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13. ABSTRACT (Maximum 200 words)

The broad purpose of this project was to investigate low-complexity interior point decomposition algorithms for stochastic programming. A specific objective was to evaluate algorithms using test problems arising from useful applications. The important direct results of this project include: (1) a new test problem collection that includes problem instances from a variety of application areas; (2) a new package of C-routines for converting SMPS input data into data structures more suitable for implementing algorithms; (3) a new software package, CPA, for two-stage stochastic linear programs. The test problems and input conversion routines have been developed in a general manner to be useful to other researchers. CPA includes volumetric center algorithms that proved to be successful in our computational evaluations. To the best of our knowledge, CPA is the only software for stochastic programming that includes volumetric center algorithms. Items (1), (2) and (3) are freely accessible over the Internet. The important theoretical results of this project include: (4) a new characterization of convexity-preserving maps; (5) a new coordinate-free foundation for projective spaces; (6) a new geometric characterization of one-dimensional projective spaces; (7) new algorithms for bound-constrained nonlinear optimization. These theoretical results are likely to be useful in computational optimization in general.

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FINAL PROGRESS REPORT

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Table of Contents

1. Introduction

This is the final progress report for ARO Grant DAAG 19-00-1-0465 awarded to the Department of Mathematics of Washington State University for the period July 24, 2000 through July 23, 2003. This grant was based on the proposal *39877-MA Low-Complexity Interior Point Algorithms for Stochastic Programming: Derivation, Analysis, and Performance Evaluation* submitted to ARO by the author.

This report is structured according to the guidelines stated in Reporting Instructions (ARO Form 18). As required in those guidelines, §4 of this report contains a list of publications and reports that acknowledge support from this grant, and §7 contains a bibliography of references. In the rest of this report we indicate references to items in this report with item number(s) within double brackets and references to items in the bibliography with item number(s) within single brackets. We shall refer to the proposal by the symbol [P].

2. Statement of the problem studied

This section follows the proposal [P] closely. The project was concerned with new, low-complexity interior point algorithms for stochastic programs including the identification of variants that are efficient in practice. We take [1] as our model

$$\begin{aligned} \text{minimize} \quad Z(x_1) &:= q_1(x_1) + \mathcal{Q}_2(x_1) \\ \text{subject to} \quad x_1 &\in \mathcal{S}_1 \end{aligned} \tag{1}$$

where $x_1 \in \mathbb{R}^{n_1}$ is the decision variable, and the feasible set $\mathcal{S}_1 \subseteq \mathbb{R}^{n_1}$ and the cost function $q_1 : \mathbb{R}^{n_1} \to \overline{\mathbb{R}}$ constitute deterministic data. The function $\mathcal{Q}_2 : \mathbb{R}^{n_1} \to \overline{\mathbb{R}}$ is defined recursively as follows: $\mathcal{Q}_t(x_{t-1}) := E[Q_t(x_{t-1}, \boldsymbol{\xi}_t)]$ is the expectation with respect to random data $\boldsymbol{\xi}_t \in \mathbb{R}^{\nu_t}$ of the value of a function Q_t for $t = 2, 3, \ldots, N$; and the dependence of function Q_t on x_{t-1} and a realization ξ_t of $\boldsymbol{\xi}_t$ is specified by

$$Q_t(x_{t-1}, \xi_t) := \inf_{x_t \in \mathbb{R}^{n_t}} \{q_t(x_t, \xi_t) + \mathcal{Q}_{t+1}(x_t) : x_t \in S_t(x_{t-1}, \xi_t)\}, \quad t = 2, 3, \ldots N - 1, \quad \text{and} \tag{2}$$

$$Q_N(x_{N-1}, \xi_N) := \inf_{x_N \in \mathbb{R}^{n_N}} \{q_N(x_N, \xi_N) : x_N \in S_N(x_{N-1}, \xi_N)\}, \tag{3}$$

where $q_t(\cdot, \xi_t) : \mathbb{R}^{n_t} \to \overline{\mathbb{R}}$ and $\mathcal{S}_t(x_{t-1}, \xi_t) \subseteq \mathbb{R}^{n_t}$, $t = 2, 3, \ldots, N$. We assume that random data $\boldsymbol{\xi}_t$ have the given discrete probability distribution

$$\{(\xi_t^l, p_t^l), l = 1, 2, \ldots, K_t\} \tag{4}$$

for $t = 2, 3, \ldots, N$, so that $\mathcal{Q}_t(\cdot) := \sum_{l=1}^{K_t} p^l Q_t(\cdot, \xi_t^l)$.

We refer to the problem defined by (1–4) as a *multistage stochastic program with recourse*. Such an optimization problem is one of the ways in which the following multistage stochastic decision-making problem illustrated in Figure 1 on p. 3 (where $\hat{K}_t := \Pi_{r=2}^t K_r$, $t = 2, 3, \ldots, N$) may be formulated for solution. The vector x_1 is a decision that has to be made at present (the stage 1). Later, at stage 2, a realization ξ_2^l of second stage random data $\boldsymbol{\xi}_2$ becomes available,

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[1] We use the symbol ':=' to indicate equality by definition in mathematical contexts, and assignment in statement of algorithms. Bold-face letters denote random variables while corresponding normal-face letters denote their realizations. Superscript T denotes transposition of vectors and matrices. Other superscripts on variables often denote indices rather than powers.

and a recourse decision x_2^l may be taken if necessary. This recourse decision is chosen from the set $\mathcal{S}_2(x_1, \xi_2^l)$ at a cost $q_2(x_2^l, \xi_2^l)$. Note that the set $\mathcal{S}_2$ depends on the decision x_1 already implemented in stage 1, and the specific realization ξ_2^l of the random variable ξ_2 observed. The decision process has a total of N stages. At each of the stages 2 through N, realizations of random variables are observed, and recourse decisions are made similarly. The formulation (1–4) states that the decision $x_1 \in \mathcal{S}_1$ that needs to be made at present *without the knowledge of realizations of random variables $\xi_2, \xi_3, \ldots, \xi_N$ that will be observed in future stages*, is to be made so that the cost $q_1(x_1)$ plus the average of the costs of future recourse decisions $\mathcal{Q}_2(x_1)$ is minimized.

There are many applications of stochastic programs with recourse. These application areas include energy planning, agriculture, airline scheduling, optimal control problems, industrial management, natural resource management such as lake eutrophication management and forestry management, telecommunication, and problems in mathematical finance such as portfolio optimization. Please consult the monographs [8, 21, 9, 3] and references contained therein. We now mention some more specialized applications. Midler and Wolmer [15] describe stochastic programming models for optimally scheduling airlift operations, devoting special attention to airlift missions among airports in the continental U.S., aerial ports of embarkation, overseas bases and aerial ports of debarkation. Martel and Al-Nuaimi [14] describe models for tactical manpower planning. A two-stage decision model for weapon acquisition with target uncertainty is described by Nickel and Mangel [17]. Refer to Manne and Richels [13] for an application related U.S. breeder reactor program. Cariño, Myers and Ziemba [6] describe a comprehensive investment, liability and risk planning model developed by Frank Russell Company for Yasuda Fire and Marine Insurance Co., Ltd. in Japan. Fragnière and Haurie [10] describe a model to identify optimal policies that the city of Geneva may adopt for meeting CO_2 emission standards required by the Federal Government of Switzerland by the year 2005.

The representation (1–4) is mathematically equivalent to the large-scale mathematical program

$$\begin{aligned}
\text{minimize} \quad Z = \quad & \hat{q}_1(x_1) + \ldots + \sum_{l=1}^{\hat{K}_t} \hat{q}_t^l(x_t^l) + \ldots + \sum_{l=1}^{\hat{K}_N} \hat{q}_N^l(x_N^l) \\
\text{subject to} \quad & x_1 \in S_1, \\
& x_t^l \in S_t(x_{t-1}^{\alpha(l,t)}, \xi_t^{\beta(l,t)}) \\
& \qquad\qquad l = 1, 2, \ldots, \hat{K}_t, \quad t = 2, 3, \ldots, N.
\end{aligned} \tag{5}$$

In (5), $\hat{K}_t = \Pi_{r=2}^t K_r$,

$$\begin{aligned}
\alpha(l,t) \quad &:= \quad \lceil l/K_t \rceil, \\
\beta(l,t) \quad &:= \quad \begin{cases} K_t & \text{if } l \bmod K_t = 0 \\ l \bmod K_t & \text{if } l \bmod K_t \neq 0, \end{cases} \\
& \qquad\qquad l = 1, 2, \ldots, \hat{K}_t, \quad t = 2, 3, \ldots, N,
\end{aligned}$$

and $\hat{q}_1(\cdot) := q_1(\cdot)$ and $\hat{q}_t^l(\cdot) := \hat{p}_t^l \cdot q_t(\cdot, \xi_t^{\beta(l,t)})$ for $l = 1, 2, \ldots, \hat{K}_t, \quad t = 2, 3, \ldots, N$ where

$$\begin{aligned}
\hat{p}_1^1 \quad &:= \quad 1, \\
\hat{p}_t^l \quad &:= \quad \hat{p}_{t-1}^{\alpha(l,t)} \cdot p_t^{\beta(l,t)}, \quad l = 1, 2, \ldots, \hat{K}_t, \quad t = 2, 3, \ldots, N.
\end{aligned}$$

This equivalence follows from the decision tree representation of (1–4) in Figure 1. In Figure 1, a path from the root to a leaf is termed a scenario. The number of scenarios is $\hat{K}_N =$
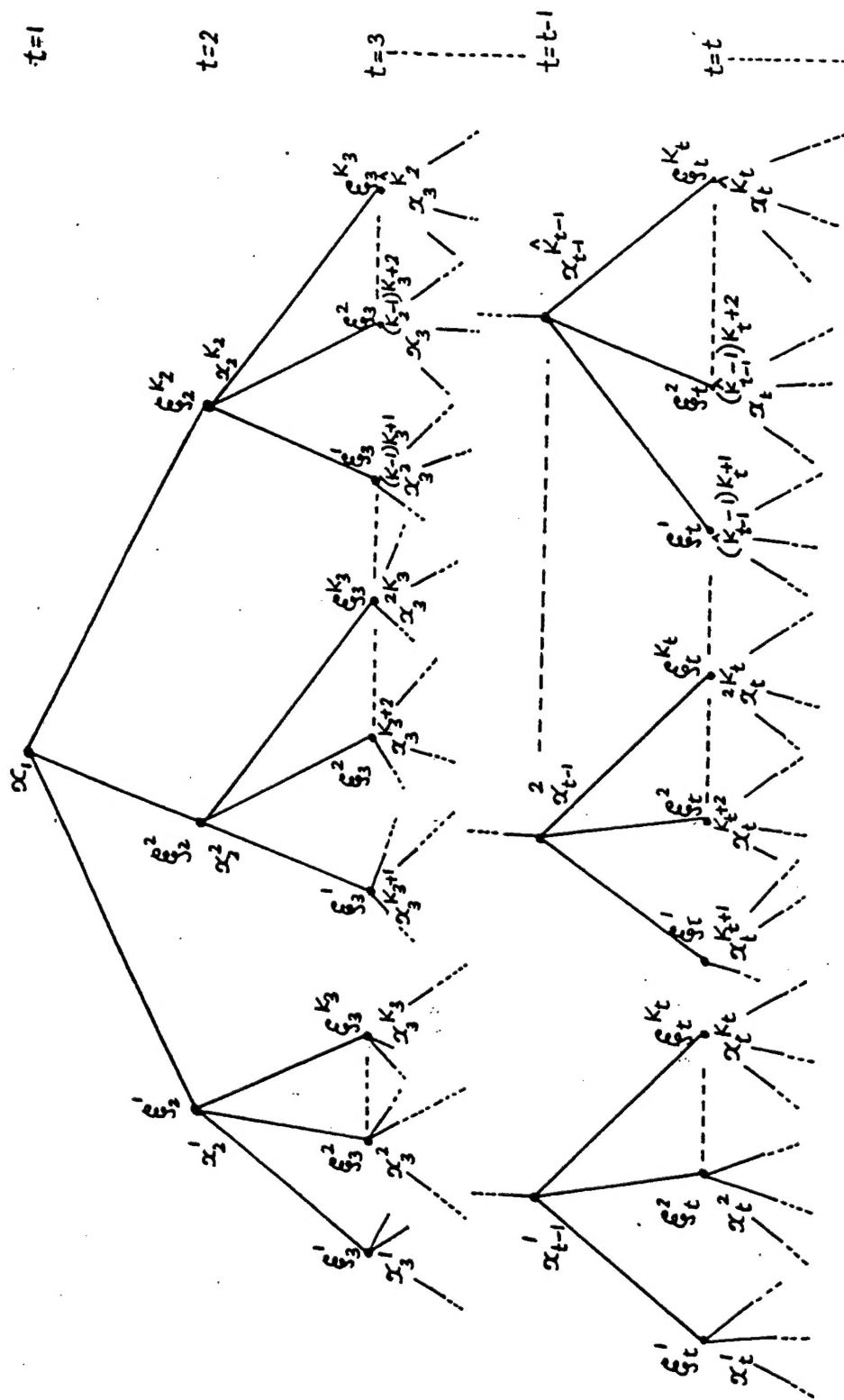
Figure 1. Illustration of a multi-stage stochastic decision problem

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$\Pi_{r=2}^N K_r$. Assume that the set $\mathcal{S}_t$ can be represented using m_t constraints. Then problem (5) has $n_1 + \sum_{t=2}^N n_t \hat{K}_t$ variables and $m_1 + \sum_{t=2}^N m_t \hat{K}_t$ constraints. Typically, even when the dimensional parameters m_t, n_t and ν_t have moderate values, problem (5) is large-scale because K_t depends exponentially on ν_t and hence K_t's and in turn $\hat{K}_t$'s are large. By way of illustration suppose that $N := 5$, and that $\nu_t := 10$, $n_t := 7$ and $m_t := 6$ for $t = 2, 3, 4, 5$. Suppose further that each of the ν_t components of the random variable ξ_t has only two possible realizations, so that $K_t = 2^{\nu_t} = 2^{10}$ for $t = 2, 3, 4, 5$. It follows that we have over a billion scenarios, and that (5) has over 7 billion variables and over 6 billion constraints. Note that, as this example illustrates, *a good indicator of the large-scale nature of a stochastic program is its number of scenarios.*

As the above example implies, practical stochastic programs are often prohibitively large for their solution by general-purpose optimization algorithms. (See for example, [16, Table V] where this is seen for the popular general-purpose code LOQO.) Fortunately, stochastic programs possess structures (note for example, the structure in the constraint matrix in (5) if the sets $\mathcal{S}_t$ could be represented by a finite number of linear constraints) that may be exploited to advantage in computational algorithms. Algorithms for stochastic programs must invariably exploit the special structures present in them. Despite tremendous advancements in the development of such special-purpose algorithms, and in computer hardware and software, computational solution of practical stochastic programs is still a challenging problem [20, 16].

Research during the past decade into interior point methods have resulted in significant advancements in the solution of deterministic linear programs. These advances are characterized by the identification of algorithms with low polynomial complexity and excellent practical performance. Motivated by this success researchers have begun to explore interior point methods for other areas of deterministic mathematical programming such as convex and semidefinite programming, and advances similar to those in the case of linear programming are being made. However, surprisingly very little research has been devoted to interior point methods for stochastic programming. In the context of stochastic programming the desirable interior point methods should again have low complexity and good practical performance. What we mean by low complexity here is complexity lower than that of a general-purpose interior point method applied on the same problem *in terms of the number of scenarios.* Indeed, such low complexity is a theoretical measure of how well the structure of the stochastic program has been exploited by the relevant special-purpose algorithm.

Against the above background, the broad purpose of the project was to derive interior point methods with such low complexity for certain important special cases of (1-4), and to identify variants with good practical performance via careful implementation and testing.

We conclude this introductory section by indicating certain special cases of (1-4) investigated in this project. *A two-stage stochastic linear program* results when $N := 2$, the functions q_1 and $q_2(\cdot, \xi_1)$ are linear, and the sets S_1 and S_2 are specified by linear constraints. *A two-stage stochastic quadratic program* (TSSQP) is similarly defined with q_1 and $q_2(\cdot, \xi)$ being quadratic instead of being linear. When $N > 2$, the functions q_1 and $q_t(\cdot, \xi_t)$ for $t = 2, 3, \ldots, N$ are linear, and the sets S_t for $t = 1, 2, \ldots, N$ are specified by linear constraints we have a *multistage stochastic linear program* (MSSLP). *A multistage stochastic convex program* (MSSCP) is similarly defined with the functions q_1 and $q_t(\cdot, \xi_t)$ for $t = 2, 3, \ldots, N$ being convex instead of being linear.

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3. Summary of the most important results

In this section we summarize the most important results of this project. Whenever possible, we do so relative to the tasks indicated in [P, pp. D-14,15,16]. Please note that we present these results in order of importance. We begin with results obtained through work performed in Tasks 2, 3 and 4.

(a) Task2

This task was on the development of a test problem collection of stochastic programs. We have developed such a test problem collection consisting of instances of TSSLP's and MSSLP's with the following specific features.

(i) At present the collection involves test problems from 11 application areas: airlift operations scheduling, forest planning, electrical investment planning, selecting currency options, financial planning, design of batch chemical plants, energy and environmental planning, network models of asset or liability management, cargo network scheduling, telecommunication network planning, and bond investment planning.

(ii) The 102-page technical report in Item [[4(e)(i)]] below describes this test problem collection in detail.

(iii) We have created a web site (http://www.uwsp.edu/math/afelt/slptestset.html) providing free access to our test problem collection. For each of the 11 families in the collection, six pieces of information are provided: description of the application and problem notation; problem statement in the same notation; numerical example (when practical); reconciliation to the standard notation for stochastic programs (as given in Item [[4(a)(iii)]] below); SMPS [4] data files for each problem instance; and optimal solution for each problem instance.

This web site is maintained by my former doctoral student Andrew J. Felt, who is now an Assistant Professor of Mathematics and Computing at University of Wisconsin, Stevens Point.

(iv) We presented details of our collection to the research community at the Ninth International Conference on Stochastic Programming held in Berlin, Germany during August 25–31, 2001. (See Item [[4(c)(iii)]] below.) Our work was well-received as fulfilling a need that existed in the field for a long time. Researchers at the meeting expressed the view that a quality test set would not only assist algorithm development by providing a standard set of problems with which to challenge new algorithm implementations, but also the teaching of stochastic programming.

To further publicize our test problem collection we prepared Item [[4(a)(iii)]] which has been accepted for publication in *INFORMS Journal on Computing*. That paper also includes a request for information on additional test problems from researchers to be cataloged in a standard manner and made available via our web site. In this sense this is an "evolving effort", and our web site will be updated as new test problem instances are received.

(v) The addition of new test problems from novel applications to our test problem collection has already begun. Martel and Al-Nuaimi [14] have presented a two-stage stochastic programming approach for optimal tactical manpower planning. Their model is based on certain assumptions that may be relaxed to make the model more relevant to applications. Cristina Cacho, a graduate student of the PI, completed a Master's

project on generalizing the model in [14] in May 2003. Six new test problem instances based on the resulting extended model will be added to the test problem collection (`http://www.uwsp.edu/math/afelt/slptestset.html`) in the near future. See Item [[4(c)(vii)]] below.

Yan Zhang, a new graduate student of the author, has just begun creating similar models for electric power system planning.

(b) Task 3

The purpose of Task 3 is to address issues necessary to implement a new family of cutting plane algorithms (CPA) for two-stage stochastic linear programs (TSSLP). In a previous paper [2], the author has shown that certain algorithms in this family of CPA's have polynomial complexity that depends linearly on the number of realizations K. This is the lowest complexity known for any computational algorithm for TSSLP's, and this family of algorithms was the first to achieve this complexity bound. Since this is a *theoretical upper bound* on the computational work required, Task 3 was to implement specific algorithms in this class so that using the test problems developed in Task 2, their *practical performance* could be assessed. We have performed the following specific work on this task.

(i) While the SMPS [4] data format (see Item [[3(a)(iii)]]) for specifying data defining the TSSLP's and MSSLP's is very convenient and natural for users, algorithms are defined using the data as specified in the mathematical form of equations (1–4) and Figure 1. We have developed a data structure suitable for implementing algorithms for TSSLP's and MSSLP's, and written C codes that converts data in the SMPS standard format into this data structure. While we developed this data structure and the conversion routines for implementing and testing *our* algorithms, this has been done in a *very general manner* so that *any algorithm* for stochastic programs can be implemented similarly.

Due to the lack of routines that convert SMPS data into a general data structure suitable for implementing algorithms for TSSLP's and MSSLP's, different input data formats are being used in different existing implementations of algorithms for TSSLP's and MSSLP's. (Indeed, we observed this when we attempted to obtain codes of existing algorithms for TSSLP's and MSSLP's for comparison with the implementation of our new algorithms.) The disadvantage is that such codes cannot directly be tested on large problem instances for which users have data only in the SMPS format.

We believe that our new data structure, and our new C routines for converting data in SMPS format into that data structure, would be used by developers of software for TSSLP's and MSSLP's. This would promote the adoption of the SMPS standard for specifying input for software for stochastic programs allowing the full realization of the benefits of the SMPS standard. It will also allow convenient testing and use of such software on large meaningful applications.

As an examination of our codes would demonstrate, developing the data structure and the conversion routines *for general use* was a major undertaking.

(ii) This work formed the Individual Project—MS in Applied Mathematics of my former Masters student Jason J. Sarich. Jason completed his MS in December 2001, and is now employed by the Mathematics and Computer Science Division of Argonne National Laboratory. The C routines are described in Item [[4(e)(iii)]].

(iii) We have created a web site (`http://www.uwsp.edu/math/afelt/slpinput.html`) providing free access to this C-routine package.

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(c) Task 4

The purpose of this task was to obtain implementations of specific algorithms in our CPA family and to assess their practical performance relative to each other as well as relative to existing codes for TSSLP's. We have developed a modular implementation of the CPA family that allows obtaining specific members of the CPA family as mentioned in Item (4.1) in [P, p. D-15]. We have named our modular software package CPA. We have also developed a parallel version of our code using PVM [11] that runs on a network of computers of the Department of Mathematics of Washington State University.

(i) Our attempts to obtain existing codes for TSSLP's as stated in [P, Item (4.2)] of the proposal have not been successful. Therefore, while we would continue to seek these codes from other researchers, we have developed modules within the CPA framework that allow obtaining an implementation of the algorithm of Van Slyke and Wets [22], the most popular existing algorithm for stochastic programming.

(ii) Using our implementation of the Van Slyke and Wets algorithm as the representative of the existing algorithms for stochastic linear programs, we have performed the comprehensive computational experiments as outlined in [P, Items (4.2),(4.3)]. In particular, we have performed experiments to compare different algorithms within CPA, to assess the effects of parallelization, to assess the use of deep cuts relative to shallow cuts, and to measure the dependence of performance on K. All these experiments have been performed using our test problem collection, and our C-routines that convert input for the problem instances in SMPS data into the data structure on which CPA was developed.

A paper detailing the development of our software package CPA and our computational experience with it will be prepared and submitted for publication shortly. At this point, we state the following two general observations on these computational results.

(iii) As indicated in [P, §3], the three new classes of algorithms that were evaluated, namely, the ellipsoid, the analytic center, and the volumetric center classes all have complexity linear in K under certain conditions. On the other hand, the complexity of Van Slyke and Wets [22] algorithm (VWA) is exponential. Thus one may expect a threshold value $\bar{K}$ such that for all $K \geq \bar{K}$ the new algorithms perform better than VWA. The important *practical* question, however, is whether $\bar{K}$ is reasonably low.

Our computational results indicate that for certain problem families in our test problem collection the value of $\bar{K}$ for the comparison of a certain member of the volumetric center class in CPA and VWA is reasonably low: this volumetric center algorithm in CPA performs better than VWA even for moderate K values. In addition, when VWA performs better, its performance is not much better than that of the volumetric center algorithm.

On the hand, the performance of the ellipsoid algorithms and the analytic center algorithms in CPA were worse than that of VWA in our experiments.

We were not surprised by the poor practical performance of the ellipsoid class: this is well-known even in the case of deterministic linear programs. However, we were surprised by the poor performance of the analytic center class, because in the deterministic case interior point algorithms based on analytic center notions are known to perform better than simplex algorithms for certain classes of practical problems.

Our observations are important, because most practical interior point algorithms for

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deterministic optimization are based on analytic center notions.

(iv) Our computational experience with the parallel version of CPA indicates that all four classes of algorithms, i.e. the three new classes of algorithms in CPA, and VWA, parallelize nearly linearly. This observation is important because on some problem instances with large K the serial version of the fastest algorithm in CPA can have running times of several hours. Thus the parallel version of CPA by virtue of its good speed-up can reduce the running time for these large problems very significantly.

(v) We have placed CPA on NEOS (Network Enabled Optimization Software) server of Argonne National Laboratory. NEOS server provides a way for users to provide input data for an optimization problem to software available on the server, have the model solved by the software, and then to have the solution reported back to them, all via the internet. See the article [19] for details. NEOS is freely accessible. Our software CPA is available at `http://www-neos.mcs.anl.gov/neos/solvers/SLP:CPA/`.

We now describe several important results obtained in this project that are indirectly related to the tasks mentioned in [P, pp. D-14,15,16]. Therefore, we label these as peripheral results, but their importance and relevance are equal to those mentioned in Items [[3(a), (b), (c)]] above.

(d) Peripheral Result 1

The field of interior point algorithms (this project investigated some of them for stochastic optimization) began with the seminal paper by Karmarkar [12] in 1984, which presented a polynomial algorithm for (deterministic) linear programs that also had excellent practical computational performance. The algorithm in that paper was derived using a map referred to as a "projective transformation". This map appeared to be strange since no other algorithm known until then (except for a little known algorithm that Davidon [7] presented in 1980) used a map more general than affine maps. Karmarkar [12] did not provide any reason for choosing this map.

In Item [[4(a)(i)]], we show that the *projective transformation of Karmarkar [12], and the map of Davidon [7] are the only continuous, injective maps that preserve convexity (of sets) in the setting of each algorithm.* Preservation of convexity is a very desirable property in the design of algorithms.

The proof of the results in Item [[4(a)(i)]] is based on a *new characterization of convexity-preserving maps* proved in Item [[4(a)(ii)]]. We were surprised when pure mathematicians indicated that our characterization is new. Therefore, Item [[4(a)(ii)]] is written in a setting much more general than that required in algorithmic work as in the present project and in Item [[4(a)(i)]].

The work in Item [[4(a)(ii)]] required a foundation for *projective spaces* not available in the pure mathematics literature. The enthusiasm we received from the referees of Item [[4(a)(ii)]], prompted us to present this foundation separately in Item [[4(d)(ii)]].

This work was performed in collaboration with Dr. William C. Davidon, whose work in the mid 1950's may arguably be considered as the beginning of the field of algorithmic nonlinear optimization.

(e) Peripheral Result 2

The Peripheral Result 1 above dealt with characterizing continuous, injective and convexity-preserving maps on subsets of vector spaces as projective maps, and demonstrating the relationships of maps used in the derivation of two seminal algorithms (Karmarkar's algorithm for linear programming [12] and Davidon's collinear scaling algorithms for nonlinear

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optimization [7]) to projective maps.

In Item [[4(d)(iii)]], we have shown that by using an apparently new notion that we term a *meridian* one can quite simply describe one-dimensional projective spaces in a geometric manner without using the sophisticated algebraic notion of a field. We were pleasantly surprised by the fact that this apparently new result in pure mathematics was motivated by our study of computational optimization algorithms.

(f) Peripheral Result 3

As mentioned in Task [P, 3.2, p. D-15], one of the technical issues that require attention in the case of volumetric center and analytic center classes of algorithms proposed in [2] is the updating of iterate x_k to x_{k+1} once a cut is added or deleted. This requires an approximate minimization of certain potential functions. While Newton's method with line searches can be used to perform this approximate minimization as suggested in [2], an attractive alternative was discussed in [1]. In [1] certain line search termination criteria are shown to be valid for strongly convex functions. While the potential functions that arise in stochastic linear programs satisfy these strong convexity assumptions, we show in Item [[4(a)(iv)]] that the line search termination criteria given in [1] would be valid under much milder assumptions: they are valid for convex functions or for strictly pseudo convex functions. However, in Item [[4(a)(iv)]] we also give a counterexample to show that these line search termination criteria may fail for nonconvex functions.

(g) Peripheral Result 4

This continues the work described in Peripheral Result 3. Specifically, we have developed new algorithms and software for bound-constrained nonlinear optimization based on Davidon's collinear scalings [7].

In Item [[4(d)(iv)]], we describe the derivation of these new algorithms and report results of our computational experiments with them. In Item [[4(d)(v)]], we report a complete convergence analysis of the new algorithms.

We now describe results obtained with work in [P, Tasks 1,5,6,7].

(h) Task 1

The purpose of this theoretical task was to extend the work described in [P, §3] for the linear case to the quadratic and convex cases thus obtaining ellipsoid, analytic center and volumetric center classes of algorithms for TSSQP's and TSSCP's.

We have derived ellipsoid, analytic center and volumetric center algorithm classes for TSSQP's and shown that they have polynomial worst case complexity linear in K. Thus the results in [P, §3] for the linear case extends to the quadratic case.

(i) Task 5

The purpose of this task is to repeat Tasks 3 and 4 on TSSLP's for TSSQP's.

The first step of this task is obtaining the analog of CPA for TSSQP's. This work is in progress.

(j) Task 6

This task (and Task 7) below are concerned with low-complexity algorithms for MSSLP's. While algorithms for two-stage problems can be extended to handle problems with more than two stages, doing so in a manner that ensures worst-case complexity *polynomial* in the number of realizations is not at all clear. The purpose of this task was to derive algorithms

that belong to the class of *direct* interior point methods (see [P, §§2.1.2, 2.2.2]). As indicated in the statement of [P, Task 6], the algorithm of Birge and Qi [5] for *two-stage* problems is a direct interior point algorithm with worst-case complexity proportional to $K^{1.5}$ where K is the number of realizations. In the statement of [P, Task 6] we conjectured that a similar algorithm could be developed for the *multistage* problem with worst-case complexity proportional to $K^{1.5}$. A crucial aspect of the algorithm of Birge and Qi [5] that results in the complexity bound proportional to $K^{1.5}$ is a procedure that computes a certain search direction in an efficient manner. We have extended that procedure to the multistage case. This extension is a major step in arriving at an algorithm with complexity proportional to $K^{1.5}$ for MSSLPs.

(k) Task 7

This task was concerned with *decomposition* interior-point algorithms for multistage problems. Note that the algorithms in Tasks 1 through 4 are decomposition interior-point algorithms for *two*-stage problems. Such algorithms for two-stage problems have complexity *linear* in K, and due to decomposition they can be implemented to exploit parallel processing directly. As indicated in the statement of Task 7, the search for such algorithms for *multistage* problem is a difficult theoretical task. We have initiated this task. Specifically, we have begun to extend the *two-stage* algorithm of Zhao [23] to the multistage case, using *volumetric barriers* instead of the *logarithmic barriers* that Zhao [23] uses. Note that logarithmic barriers form the basis for analytic centers, and our computational experience with the two-stage case (see Item [[3(c)(iii)]] above) indicates that volumetric center algorithms perform much better in practice than analytic center algorithms.

Yuntao Zhu, a new doctoral student of the author, has just begun exploring these ideas as part of his doctoral dissertation research.

4. Listing of all publications and technical reports supported under this grant

Copies of all these papers have been submitted to ARO according to Reporting Instructions (ARO Form 18) and are available from the author (`email: ari@wsu.edu`). Manuscripts submitted for publication that have not yet appeared in the open literature are also available on the web at `http://www.wsu.edu:8080/~ari/preprints.html`.

(a) Papers published in peer-reviewed journals

(i) K. A. Ariyawansa, W. C. Davidon and K. D. McKennon, 'On a characterization of convexity-preserving maps, Davidon's collinear scalings and Karmarkar's projective transformations', *Mathematical Programming* (1) 90 (2001) 153–168.

(ii) K. A. Ariyawansa, W. C. Davidon and K. D. McKennon, 'A characterization of convexity-preserving maps from a subset of a vector space into another vector space' *Journal of the London Mathematical Society* (2) 64 (2001) 179–190.

(iii) K. A. Ariyawansa and A. J. Felt, 'On a new collection of stochastic linear programming test problems', *INFORMS Journal on Computing* (accepted for publication and in press).

(iv) K. A. Ariyawansa and W. L. Tabor, 'A note on line search termination criteria for collinear scaling algorithms', *Computing* 70 (2003), 25–39.

(b) **Papers published in non-peer-reviewed journals or in conference proceedings**

None.

(c) **Papers presented at meetings but not published in conference proceedings**

(i) On a characterization of convexity-preserving maps (17th International Symposium on Mathematical Programming, Atlanta, GA, August 7-11, 2000)

(ii) A. J. Felt, K. A. Ariyawansa and J. J. Sarich, 'Open source input routine for SMPS data', INFORMS International Meeting, Maui, Hawaii, June 17–20, 2001.

(iii) A. J. Felt and K. A. Ariyawansa, 'A collection of multistage stochastic linear programming test problems', Ninth International Conference on Stochastic Programming, Berlin, Germany, August 25–31, 2001.

(iv) K. A. Ariyawansa and A. J. Felt, 'Recent numerical experience with an implementation of the volumetric center cutting plane algorithm for stochastic programming', Seventh SIAM Conference on Optimization, Toronto, Canada, May 20–22, 2002.

(v) K. A. Ariyawansa, A. J. Felt, J. J. Sarich and R. O'Fallon, 'On the computational performance of a volumetric center algorithm for stochastic programming', INFORMS National Meeting, San Jose, CA, November 17–20, 2002.

(vi) K. A. Ariyawansa, 'New algorithms with polynomial complexity for a class of stochastic optimization problems', Presentation for the Board of Visitors, Department of Mathematics, Washington State University, Pullman, WA October 3, 2002.

(vii) Cristina Cacho, K. A. Ariyawansa and Andrew J. Felt, 'On the construction of a family of stochastic programming test problems', INFORMS National Meeting, Atlanta, GA, October 19–22, 2003.

(d) **Manuscripts submitted, but not published**
All these manuscripts are presently under consideration by the peer-reviewed journals indicated.

(i) M. Vaziri, K. A. Ariyawansa, K. Tomsovic and A. Bose, 'On the accuracy of approximating a class of NP-complete quadratic programs (QP) by a class of polynomially bounded linear programs (LP)' (submitted to *IEEE Transactions on Circuits and Systems*).

(ii) K. A. Ariyawansa, W. C. Davidon and K. D. McKennon, 'A coordinate-free foundation for projective spaces treating projective maps from a subset of a vector space into another' (submitted to *Dissertationes Mathematicae*).

(iii) K. A. Ariyawansa, W. C. Davidon and K. D. McKennon, 'One-dimensional projective space: Avatar of a meridian' (submitted to *Journal of Geometry*).

(iv) K. A. Ariyawansa and W. L. Tabor, 'A class of collinear scaling algorithms for bound-constrained optimization: Derivation and computational results' (submitted to *Optimization Methods and Software*).

(v) K. A. Ariyawansa and W. L. Tabor, 'A class of collinear scaling algorithms for bound-constrained optimization: Convergence theorems' (submitted to *Optimization*).

(e) **Technical reports submitted to ARO**
This list contains technical reports (not intended for journal publication), Master's project reports, and doctoral dissertations.

(i) A Collection of Multistage Stochastic Linear Programming Test Problems (Version 1), Technical Report 00-3, Department of Pure and Applied Mathematics, Washington State University, Pullman, WA 99164-3113 (This reports the present status of an evolving effort; see Item [[3(a)(iv)]] above.)

(ii) J. J. Sarich, 'Expanding the Capabilities of the TAO Optimization Package', Group Research Project—M.S. in Applied Mathematics, Department of Mathematics, Washington State University, December 2001.

(iii) J. J. Sarich, 'A C Implementation of an Input Routine for Two-Stage Stochastic Linear Programs', Individual Project Report—M.S. in Applied Mathematics, Department of Mathematics, Washington State University, December 2001.

(iv) Wayne L. Tabor, 'Collinear Scaling Algorithms for Optimization Using Line Searches and Trust Regions', Ph.D. Dissertation in Mathematics, Department of Mathematics, Washington State University, Pullman, WA, December 2002.

5. List of all participating scientific personnel

(a) K. A. Ariyawansa, Professor of Mathematics, Washington State University, Pullman, WA (Principal Investigator)

(b) A. J. Felt, Assistant Professor of Mathematics and Computing, University of Wisconsin-Stevens Point, Stevens Point, WI (former doctoral student of the P.I. at Washington State University, participated in this project but was not supported by this grant).

(c) Jason J. Sarich, Research Associate, Mathematics and Computer Science Division, Argonne National Laboratory, Argonne, IL (participated in this project, was supported by this grant as a research assistant, and received the MS in Applied Mathematics from Washington State University in December 2001).

(d) Wayne L. Tabor, Visiting Assistant Professor of Mathematics, Washington State University (former doctoral student of the P.I., participated in this project but was not supported by this grant, and received the Ph.D. in Mathematics from Washington State University in December 2002).

(e) Cristina Cacho, GE Capital Bank, Mexico City, Mexico (participated in this project but was not supported by this grant, and received the MS in Mathematics from Washington State University in May 2003).

(f) Ryan O'Fallon, graduate student, Washington State University, Pullman, WA (participated in this project, supported by this grant as a research assistant, and is expected to receive the MS in Mathematics from Washington State University in December 2003).

6. Report of inventions
In this section we list the following three "technology transfer" items.

(a) A new stochastic programming test problem collection with free access at http://www.uwsp.edu/math/afelt/slptestset.html (see Item [[3(a)]] above).

(b) A new C-routine package for converting SMPS [4] input data into data structures suitable for implementing algorithms. This package is freely available at `http://www.uwsp.edu/math/afelt/slpinput.html` (see Item [[3(b)]] above).

(c) A new software package CPA for two-stage stochastic programming. This package can be used freely via the NEOS [19] server (`http://www-neos.mcs.anl.gov/neos/solvers/SLP:CPA/`) of the Argonne National Laboratory. See Item [[3(c)]] above.

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